BRITTLE STRENGTH OF VESSELS UNDER PRESSURE

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The problem of brittle strength has become very important in connection with the extensive application of large vessels under pressure in the atomic, missile, and chemical industries. The first successes in the solution of this problem, based on linear fracture mechanics, also showed the directions for the subsequent studies [1].

In the following we examine some aspects of linear and nonlinear fracture mechanics in application to thinwall vessels made from high-strength materials. We first present briefly the fundamentals of the engineering method for brittle strength analysis and indicate the primary factors which have not been studied. We then investigate the superfine structure of the crack end and develop a fracture theory relating to phenomena of smaller scale [2] (Section 1). In Section 2 we study the effect of loading rate on fracture toughness. In Section 3 we evaluate brittle strength for the case of an elliptic defect with account for residual stresses. In conclusion, we touch on the questions of reliability (Section 4).

The brittle strength of a thinwall vessel is determined by the shape and the location of the most hazardous cracklike defect and the magnitude of the fracture toughness K_{1*}, which characterizes the intensity of the elastic stresses near the crack edge at the moment when its unstable growth starts (the notation K_{IC} is usually used for this quantity [1]). The metallurgical and technological causes for the formation of cracklike defects are discussed in detail in [3].

In accordance with linear fracture mechanics, the procedure for brittle strength analysis consists in elastic stress analysis for the body with a cut of specified form; then the maximal stress intensity factor at the crack contour is equated to the quantity K_{1*}, which is assumed constant and known from a specially posed experiment. Within the framework of linear analysis, in addition to recording the initial cracklike defect and measuring K_{1*}, primary attention must be devoted to study of strength nonhomogeneity and anisotropy, and also the residual stresses.

Most critical are the embrittled thermal influence zones near weld seams. Of all the material mechanical characteristics K_{1*} is most structure sensitive. Therefore, for the same chemical composition it depends, for example, on the rolling direction, heat treatment, smelting, and so on. In the most general case K_{1*} is a function of the three Cartesian coordinates (nonhomogeneity) and the three parameters which define the position of the crack edge at a given point (anisotropy). As a rule, the material can be considered homogeneous and isotropic with respect to the elastic properties.

However, effects which cannot be explained theoretically within the framework of linear fracture mechanics are still more interesting. The most important of these from the practical viewpoint are associated with subcritical crack growth [4] and the large relative size of the plastic zones at the tip of the crack [1, 5].

1. SUPERFINE STRUCTURE OF CRACK END

We denote by L the characteristic linear dimensions of the body (crack length or distance from crack tip to the edge of the body), d denotes the characteristic linear dimension of the plastic region in the critical state, and ρ is the characteristic linear dimension of the crack tip in the limiting state (for example, the radius of curvature or aperture of the crack). Strictly speaking, linear fracture mechanics is applicable provided $L \gg d$ (when the dimensionless parameter $\chi = K_{1*}^2 \sigma_s^{-2} L^{-1}$ is considerably smaller than one [6];

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here σ_S = yield point). When this condition is violated, the elastic singularity is not realized and its use is meaningless. The latter case is more realistic for constructional metals of low and moderate strength and the vessel thicknesses usually used.

Let us examine the region at distances r from the crack edge, satisfying the conditions

$$\rho \ll r \ll d, \qquad \rho \ll r \ll L \tag{1.1}$$

For the structural metals, the quantity d is at least two to four orders larger than ρ; therefore, the admissibility of one condition causes no question. The other condition is violated only for cavities. The solution of the problem of the stress and deformation distributions in the region (1.1) provides the answer to the question of the crack-end superfine structure.

Within the framework of small deformation theory the quantity ρ for a cut will obviously be equal to zero.

We shall consider the material to be strain-hardening elastoplastic. We shall use deformation theory, assuming the loading of each element nearly simple. The admissibility of this assumption is also confirmed later by the nature of the solution obtained.

We write the basic relations [7]:

equilibrium equation

$$\sigma_{ij,j} = 0 \qquad (i,\ j = 1,\ 2,\ 3) \tag{1.2}$$

relations between deformations and displacements

$$\varepsilon_{ij} = {}^1/_2\,(u_{i,j} + u_{j,i})$$

Hencky equations

$$\varepsilon_{ij} = \frac{f(I)}{2I}\sigma_{ij} - \frac{1}{3}\sigma_{ii}\delta_{ij}\left[\frac{f(I)}{2I} - \frac{1-2\nu}{E}\right] \tag{1.3}$$

$$\Gamma = f(I), \qquad I = \sqrt{[\sigma_{ij} - {}^1/_3\,\sigma_{ii}\delta_{ij}]\,[\sigma_{ij} - {}^1/_3\,\sigma_{ii}\delta_{ij}]}, \quad \Gamma = 2\sqrt{[\varepsilon_{ij} - {}^1/_3\,\varepsilon_{ii}\delta_{ij}]\,[\varepsilon_{ij} - {}^1/_3\,\varepsilon_{ii}\delta_{ij}]}$$

Here σ_{ij} = stresses, ε_{ij} = deformations, u_i = displacements, ν = Poisson coefficient, E = Young's modulus, and $f(I)$ = given strain-hardening function, satisfying the condition $f'(I) > 0$. Moreover, we shall assume that the strain-hardening function approaches asymptotically the linear form

$$f(I) = \frac{I - I_0}{\mu_0} \quad \text{for} \quad I \to \infty \tag{1.4}$$

Here μ_0 and I_0 are constants of the material. It can be shown that the closed system of equations (1.2) and (1.3) is elliptic if the condition $f'(I) > 0$ is satisfied.

We examine a small vicinity of the arbitrary point O on the crack contour; we take the point O as the origin of the x_i Cartesian coordinates; and we direct x_3 along the crack contour and x_2 along the normal to the crack surface, which is free of loads. Making the passage to the limit which is equivalent to the "microscope principle," we obtain the canonical singular problem for (1.2), (1.3), given outside the cut $x_2 = 0$, $x_1 < 0$. In this case we must set in (1.2), (1.3)

$$\frac{\partial}{\partial x_3} = 0, \qquad \frac{f(I)}{2I} = \frac{1}{2\mu_0} \tag{1.5}$$

The second condition is a consequence of the stress singularity as $x^2 + y^2 \to 0$, and also the positive definiteness of the quantity I and the relation (1.4). It can be shown that a solution of the posed problem which is bounded (as $x^2 + y^2 \to 0$) in the stresses and continuous does not exist (in particular, this follows from energy considerations [2]).

Thus, the superfine crack-tip structure for the material in question with asymptotic linear strain-hardening coincides with the fine crack-end structure for a linearly-elastic material having the elastic constants

$$E_0 = 2\mu_0(1 + \nu_0), \qquad 2\nu_0 = \frac{E - 2\mu_0(1 - 2\nu)}{E + \mu_0(1 - 2\nu)} \tag{1.6}$$

Here E_0 is Young's modulus, and ν_0 is the Poisson coefficient. We denote the corresponding stress intensity factors by k_1, k_2, k_3. We recall that the fine crack-end structure is defined by the relation $d \ll r \ll L$.

Now we can use any of the numerous models suggested previously [8-14] to formulate the local fracture condition. All these models are equivalent [15] and lead to the same formulation of the criterion in terms of stress intensity factors, first given by Irwin [16]. We note that among the cited authors McClintock and Wells proposed their criteria specifically for the superfine structure.

We present the formulation of our additional condition on the contour of a growing normal-discontinuity crack

$$k_1 = k_{1*}, \quad k_1 = \lim_{x_1 \to 0} (\sqrt{2\pi x_1}\, \sigma_{22}) \tag{1.7}$$

Here k_{1*} is a material constant, related very simply with another important characteristic of the material

$$k_{1*}^2 = 2 E_0 \gamma / (1 - \nu_0^2) \tag{1.8}$$

Here γ is a quantity having the dimensions of specific surface energy; in the considered elastoplastic model of the medium, it is equal [15] to the work of the finite plastic deformations immediately near the crack edge in a layer having a thickness on the order of the radius of curvature of the crack at its tip (for the metals 10^{-5}-10^{-2} cm). Within the framework of the small deformation theory adopted here, this quantity is obviously not taken into account in the model.

The case of asymptotic power-law strain-hardening function can be examined similarly:

$$f(I) = 2a I^{\varkappa+1} \text{ for } I \to \infty \tag{1.9}$$

Here a and $\varkappa$ are constants of the material. In this case it is not difficult to show, using [2], that the additional condition on the contour of the growing normal-discontinuity crack will be the following:

$$k_1 = k_{1*}, \quad k_1 = \lim_{x_1 \to 0} [(2\pi x_1)^{1/(\varkappa+2)} \sigma_{22}] \tag{1.10}$$

Here the constant k_{1*} is connected with the quantity γ by the relation

$$k_{1*}^{\varkappa+2} = \lambda_1 \gamma / a \tag{1.11}$$

where λ_1 is a dimensionless function of $\varkappa$.

On the basis of this discussion, the basic problem of nonlinear fracture mechanics for a strain-hardening body during loading is posed as follows: we are required to solve (1.2), (1.3) in the region occupied by the body, satisfying the boundary conditions at the surface of the body (and cracks) and the additional condition (1.7) or (1.10) at the contour of the growing normal-discontinuity crack. For the loading in this case, it is required that in the superfine structure the increment Δk_1 be positive. Additional studies are required in the case of a complex loading path with unloading, since the Hencky equations cannot be used and Prandtl–Reuss theory must be used; difficulties arise with determining the residual stresses and strains.

We see that the introduction of strain-hardening makes it possible to avoid successfully the difficulties characteristic of the ideally elastoplastic body (see the article by Irwin and McClintock in [1], and also [2]), which amount to the fact that in the general case the stress and strain distribution near the edge of a cut in such a body cannot in principle be represented by a finite number of undetermined constants. This leads to a possible divergence between the different local fracture criteria for the hyperfine structure. Naturally, in the case in which the body dimensions are sufficiently large so that the quasibrittle asymptotic behavior is reached, all these criteria are equivalent for the ideally elastoplastic model as well.

Let us make some estimates. Here we shall suppose for simplicity that the strain hardening function in the entire region $I > \sigma_{0.2}/\sqrt{3}$ is approximated by the power-law relation (1.9). We should emphasize that the relation $\Gamma = f(I)$ must be determined on sufficiently thin smooth specimens.

At a distance from the crack point of the order of its opening ρ, the stress σ_{22} is on the order of the engineering ultimate strength of the thin smooth specimen. Hence, with the aid of (1.10) we obtain

$$k_{1*}^{\varkappa+2} \approx 2\pi\rho\sigma_b^{\varkappa+2} \tag{1.12}$$

At a distance from the crack point of the order of the plastic region dimension d in the critical state (the quasibrittle state is assumed to have been reached), the stress σ_{22} is on the order of the $\sigma_{0.2}$ yield point. Hence, with the aid of (1.10) we find approximately

$$k_{1*}^{\varkappa+2} \approx 2\pi d \sigma_{0.2}^{\varkappa+2} \tag{1.13}$$

Using the known relations [1, 6]

$$K_{1*}^2 \approx 2\pi\sigma_{0.2}^2 d, \qquad K_{1*}^2 = 2E\gamma_* / (1 - \nu^2) \tag{1.14}$$

and (1.11)-(1.13), we find some interesting relations:

$$k_{1*}^{\varkappa+2} = \lambda_2 \sigma_{0.2}^{\varkappa} K_{1*}^2, \quad \rho = \lambda_3 \frac{\sigma_{0.2}^{\varkappa} K_{1*}^2}{2\pi\sigma_b^{\varkappa+2}}, \qquad \frac{\rho}{d} = \lambda_4 \left(\frac{\sigma_{0.2}}{\sigma_b}\right)^{\varkappa+2} \tag{1.15}$$

where γ_* is the total specific dissipation work (effective surface energy per unit area), and λ_2, λ_3, λ_4 are numbers of order one.

It also follows from these estimates that the quantities γ and γ_* are of the same order, i.e., in the process of quasibrittle crack development the irreversible specific work on finite deformations immediately near the crack edge (at distances from the edge less than a quantity of order ρ) amounts to a significant portion of the total specific dissipation energy in contrast with the case of the plane stress state of very thin plates [17]. This explains the experimental observation that subcritical crack growth for plane strain is far less than for very thin plates. On the basis of fractographic studies of fatigue crack surfaces (see, for example, the article of Beecham and Pell in [1]), we can conclude that subcritical crack growth under plane strain conditions is of the order of magnitude of the opening ρ, while in very thin plates this growth may be measured in centimeters [18].

The approach proposed may be used, in particular, to measure fracture toughness K_{1*} on small specimens with a crack by measuring k_{1*}. To do this we must have the theoretical solutions for the corresponding geometric configuration and for a semi-infinite cut in an infinite body; the latter solution yields the exact dependence between K_{1*} and k_{1*}. The use of digital computers makes it possible to hope that these problems will be solved in the near future.

We shall examine a concrete example. Assume a crack of length l perpendicular to the surface runs to the edge of a half-plane (plane strain). At infinity the body is subjected to the uniform tensile stress p; the surface of the body and crack are assumed free of loads. We take the above power-law approximation (1.9) for $I \geq \sigma_{0.2}/\sqrt{3}$. Using dimensional analysis [10], it is not difficult to find the magnitude of the fracturing stress p_* in the limiting cases:

$$p_* = \lambda_5 K_{1*} l^{-1/2} \quad \text{for} \quad \chi \ll 1 \tag{1.16}$$

$$p_* = \lambda_6(\varkappa)\, k_{1*}\, l^{-1/(\varkappa+2)} \quad \text{for} \quad \chi \gg 1$$

$$(\chi = K_{1*}^2\, l^{-1}\, \sigma_{0.2}^{-2}) \tag{1.17}$$

Here λ_5 is a number, $\lambda_6(\varkappa)$ is a dimensionless function of $\varkappa$; in the case of a specimen of finite width h they also depend on the ratio l/h. The second formula is obviously valid for $l \gg \rho$; when the crack length becomes comparable with ρ, the crack grows like a cavity and $p_* = \sigma_b$.

2. EFFECT OF LOADING RATE ON FRACTURE TOUGHNESS

The fracture toughness index K_{1*} also varies as a function of the loading rate (by approximately 1.5–2 times with a five-order change of the loading rate, which corresponds to transition from static to impact loading; see, for example, the article of Irwin and Krafft [1]). Two basic types of physical mechanisms for such a dependence can be suggested (we neglect slow initial subcritical crack growth).

a) Local Ageing [20]. Let us assume that the vicinity of the crack point is subjected to loading of magnitude K_1, less than critical. In the region near the crack point, under the action of the stresses the phase changes, recrystallization processes, and so on, and also the diffusion processes (e.g., adsorption of hydrogen from the ambient medium), take place faster; and the hydrogen diffusion rate in the crack, in view of the high volatility of hydrogen, can be considered infinitely large in comparison with the diffusion in the solid body. All such locally proceeding processes, thermally activated, can be described phenomenologically as a modification, "ageing," of the material at the crack tip in the course of time. The materials having such a physical mechanism include certain titanium alloys [1].

In this case, using the conventional fluctuation arguments and the damage summation law, it is not difficult to obtain the following relation for the time τ at the end of which the crack transitions into the unstable state:

$$\int_0^\tau \exp \frac{\eta K_1(t)}{kT} dt = \tau_0 \exp \frac{U}{kT} \tag{2.1}$$

Here T is temperature, k is the Boltzmann constant, and τ_0, η, and U are constants of the material.

In particular, for constant rate $K_1^{\cdot}$ = const, when $K_1 = K_1^{\cdot} t$, from (2.1) we obtain the dependence of the fracture toughness K_{1*} on the loading rate $K_1^{\cdot}$:

$$K_{1*} = \frac{kT}{\eta} \ln\left(1 + \frac{\tau_0 \eta}{kT} K_1^{\cdot} \exp \frac{U}{kT}\right) \tag{2.2}$$

Since the second term in the parentheses is much larger than 1, we obtain the following formula:

$$K_{1*} = \tau_0 K_{10}^{\cdot} \left(\frac{U}{kT} + \ln \frac{K_1^{\cdot}}{K_{10}^{\cdot}}\right) \qquad \left(K_{10}^{\cdot} = \frac{kT}{\tau_0 \eta}\right) \tag{2.3}$$

which describes quite well [1] the experimental results for the Ti–6Al–4V.

b) Yield Lag. (Cf. [21] and [22].) Let us assume that the fine structure of the crack point is subjected to loading with the constant rate $K_1^{\cdot}$ = const, so that $K_1^{\cdot} = K_1 t$. As a result of the plastic deformation time lag, the larger the loading rate $K_1^{\cdot}$, the smaller the plastic region near the crack point at any fixed moment of time.

In the low-carbon steels, which have the yield lag property, there is some minimal value of K_{1*} for a given temperature, reached on a stationary crack during dynamic testing or a running crack at the moment of stopping [1]. The descending segment on the $K_{1*} = f(K_1^{\cdot})$ curve is quite well described by (2.3), if therein we take the minus sign on the logarithm. There is not as yet a satisfactory theory for this phenomenon. The indicated dependence on the loading rate explains the abrupt crack development observed in certain metals.

3. BRITTLE STRENGTH COMPUTATION FORMULAS

ACCOUNT OF RESIDUAL STRESSES

In addition to other factors, the brittle strength of vessels depends significantly on the residual stresses, which develop primarily during manufacturing operations. Particularly hazardous are the internal stresses in the welding heat affected zone, which have a local nature and reach considerable magnitude. Account for these stresses within the framework of linear fracture mechanics obviously leads to mathematical problems of elasticity theory without initial stresses but with an external load distributed along the surface of the crack.

Assume that a plane part-through crack which in plane view is a semiellipse with axes a and b (Fig. 1) runs to the edge of the vessel wall. We shall consider that the structure has failed if as a result of growth the crack becomes a through crack (even if this does not lead to failure of the entire vessel). For this problem in the usual approximation of shell theory ($h \ll R$, where h is the wall thickness and R is the smallest radius of curvature of the shell), the vessel wall may be considered an infinite strip $0 \le y \le h$, $-\infty < (x, z) < \infty$, whose boundaries $y = 0$ and $y = h$ are free of loads; at infinity there act tensile stresses, bending and twisting moments, determined from the solution of the problem as a whole for the subject shell without a crack. They are equal to the corresponding quantities from the analysis of the shell at that spot where the crack is located and depend on the geometric parameters of the shell, internal pressure, and other external loading possible parameters.

We shall consider the problem to be locally symmetric relative to the xy plane (Fig. 1); in this case only the stress $\sigma_z = \sigma_0$ and the bending moment $M_x = M_0$ will influence the crack growth. We assume that in the absence of the crack there existed initial stresses at its location, which we approximate by a linear relation

$$\sigma_z = c_0 y + d_0, \qquad \tau_{zx} = \tau_{zy} = 0 \tag{3.1}$$

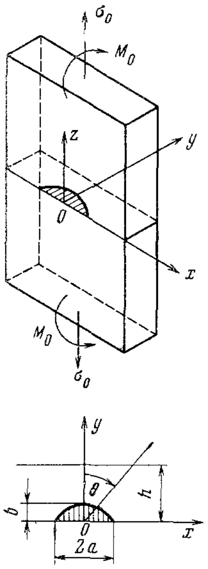

Fig. 1

It is not difficult to show that in this case the unknown stress intensity factor K_1 is determined from the solution of the analogous problem for a strip with crack, free of external loads, with the conditions at infinity

$$\sigma_z = \sigma_0 - d_0, \qquad M_x = M_0 - {}^1/_{12}\, c_0 h^3 \tag{3.2}$$

From considerations of dimensional analysis and problem linearity, this factor will be expressed by the formula

$$\begin{aligned} K_1 &= (\sigma_0 - d_0)\sqrt{b}\, \varphi\,(\theta,\ b\,/\,a,\ b\,/\,h) \\ &+ (M_0 - {}^1/_{12}\ c_0 h^3) b^{-3/2} \psi\ (\theta,\ b\,/\,a,\ b\,/\,h) \end{aligned} \tag{3.3}$$

where φ and ψ are dimensionless functions of their variables.

In order to obtain a sufficiently simple engineering solution of this problem, we shall use the available exact solutions.

We shall utilize the approximate estimate techniques whose effectiveness was demonstrated in studies of Irwin [23] and Paris and Sih [1].

We shall examine the range

$$0 \leqslant b\,/\,a \lesssim 1, \quad 0 < b\,/\,h \lesssim 0.5 \tag{3.4}$$

which is of greatest practical interest.

Let us first examine some particular cases which are also of independent interest.

Crack of Elliptic Planform in Infinite Body. Let an infinite body with crack occupying the region $z = 0$, $x^2/a^2 + y^2/b^2 \leq 1$ be subjected to uniform tension in the direction of the z axis by the stress σ at infinity. The stress intensity factor K_1 is expressed by the following approximate formula:

$$K_1 = \sigma \sqrt{\pi b}\,(1 - 0.36\ b\,/\,a)\ [\cos^2\theta + (b^2\,/\,a^2)\sin^2\theta]^{1/4}, \quad (0 \leqslant b\,/\,a \leqslant 1,\ x = a\ \sin\theta) \tag{3.5}$$

coinciding to within 1% with the exact Panasyuk–Irwin formula [1], which contains an elliptic integral.

On the basis of (3.5) the maximum of K_1 occurs for $\theta = 0$ at the ends of the ellipse short axis. In this case, if $0.78 < b/a < 1$ the initiation of rapid unstable growth of the entire crack is preceded by slow stable initial growth of the brittle crack, in the process of which the crack shape approaches a circle with diameter equal to the length $2a$ of the large axis of the initial ellipse (if we assume that the crack retains in its growth an elliptic form). The circular crack form $x^2 + y^2 \leq a^2$ corresponds to the moment of transition into the unstable state. In this case the limit load σ_*, corresponding to fracture of the body as a whole, is found from (3.5) for $a = b$ and from the condition $K_1 = K_{1*}$

$$\sigma_* = \frac{1}{2} \sqrt{\pi} K_{1*} a^{-1/2} \qquad (0.78 < b\,/\,a < 1) \tag{3.6}$$

Formulas (3.6) and (3.5) can be recommended as design formulas for vessels under pressure if the shortest distance of the contour points of the most hazardous crack-like defect from the vessel wall or neighboring defect is no less than $3a$ [1].

We note that the formula proposed by Irwin [23] applies only to the case of unstable growth of an elliptic crack.

In the case in which a normal load following the linear law

$$\sigma_z = -12\ My\ h^{-3} \tag{3.7}$$

is applied to the edges of a circular crack $z = 0$, $x^2 + y^2 \leq a^2$ in an infinite body, the stress intensity factor K_1 will be [24]

$$K_1 = 16\ \pi^{-1/2}\ M a^{3/2}\ h^{-3} \cos\theta \tag{3.8}$$

Superposition of (3.6) and (3.8) makes it possible to estimate the fracturing load for bending-tension of a flat plate in those cases in which $K_1 > 0$ everywhere on the contour of a circular crack located at a distance no less than $3a$ from the edges of the strip. In this case crack growth will precede the moment of transition into the unstable state.

This stable initial growth of the brittle crack in a uniform stress field will be the characteristic feature of three-dimensional cracks.

Semicircular Surface Crack in a Strip. Let a body occupying the region $0<y<h$, $-\infty<(x, z)<\infty$ and having a crack $z=0$, $x^2+y^2 \le a^2$ be subjected at infinity to the uniform tensile stress $\sigma_z=\sigma$ and the bending moment $M_x=M$. The stress intensity factor K_1 for bending is expressed by the approximate formula

$$K_1 = 6.8Mh^{-3/2}\sqrt{\frac{a}{h}}\left[1-1.4\frac{a}{h}+\left(\frac{2\theta}{\pi}\right)^2\left(0.2+\frac{a}{h}\right)\right] \tag{3.9}$$
$$(0<a/h\lesssim 0.5,\ \sin\theta=x/a)$$

which coincides to within about 3% with the numerical solution of [25].

Using the solution of [25], to within the same accuracy we can find the approximate expression for the stress intensity factor for tension

$$K_1=\frac{2}{\sqrt{\pi}}\sigma\sqrt{a}\left[1+0.2\left(\frac{2\theta}{\pi}\right)^2\right] \qquad (0<a/h\lesssim 0.2,\ \sin\theta=x/a) \tag{3.10}$$

By analogy with the preceding discussion, the crack obviously will initially grow stably from the edges adjacent to the free boundary, taking an elliptic shape, until the stress intensity becomes constant along the entire contour of the crack. Then the unstable dynamic fracture process begins. On the basis of experimental data [25], the ellipse axis ratio a/b at the moment of tension fracture is about 1.5.

Rectilinear Surface Crack in a Strip. Let a body occupying the region $0<y<h$, $-\infty<(x, z)<\infty$ and having a crack $z=0$, $0<y\le b$ be subjected at infinity to uniform tension by the stess $\sigma_z=\sigma$ and the bending moment $M_x=M$.

The corresponding stress intensity factors will be

$$K_1=\sigma\sqrt{\pi b}\,\frac{1.11+5(b/h)^4}{1-b/h} \qquad \left(0<\frac{b}{h}\lesssim 0.5\right) \tag{3.11}$$

$$K_1=\frac{4.2\lambda M}{h^{3/2}}\left[\left(1-\frac{b}{h}\right)^{-3}-\left(1-\frac{b}{h}\right)^3\right]^{1/2} \qquad \left(0<\frac{b}{h}<1\right)$$
$$\lambda=\begin{cases}1.15-15(b/h)^2 & \text{for } 0<b/h<0.1\\ 1 & \text{for } b/h>0.1\end{cases} \tag{3.12}$$

Formula (3.11) is an approximation to within 1% of the numerical results of Gross and Bowie (presented in [1]). Expression (3.12) is the modified Romaine formula; comparing it with Bueckner's numerical solution, its maximal error can be found to about 4%. We note that all these results were confirmed experimentally using Irwin's method (measuring the displacement or compliance).

These schemes and the formulas (3.11), (3.12) are the most convenient in practice for measuring the K_{1*} of metals.

Let us return to the general case of an elliptical edge crack in a strip (Fig. 1). Using the formulas obtained above as different limiting estimates, we can find the following approximate expressions for the stress intensity factor:

tension

$$K_1=\sigma\sqrt{\pi b}\times\frac{1.12-0.48b/a+0.13(2\theta/\pi)^2(b/a)(3b/a-2-b/h)}{1-(b/h)(1-0.75b/a)}$$
$$(0<b/a<1,\ 0<b/h\lesssim 0.4) \tag{3.13}$$

bending

$$K_1=6.8Mh^{-3/2}\left(\frac{b}{h}\right)^{1/2}\left\{\frac{b}{a}\left(\frac{2\theta}{\pi}\right)^2\left(\frac{b}{h}-0.4+0.6\frac{b}{a}\right)\right.$$
$$\left.+\frac{b}{a}\left(1-1.4\frac{b}{h}\right)+0.62\lambda\left(1-\frac{b}{a}\right)\left(\frac{b}{h}\right)^{-1/2}\left[\left(1-\frac{b}{h}\right)^{-3}-\left(1-\frac{b}{h}\right)^3\right]^{1/2}\right\}$$
$$(0<b/a<1,\ 0<b/h\lesssim 0.4) \tag{3.14}$$

Formulas (3.13), (3.14) include as limiting relations (3.9)-(3.12); on the basis of physical arguments and the indicated precision of the limit formulas, we can guarantee that the maximal error will not exceed 10%. This accuracy should be considered satisfactory for engineering purposes. Superposition makes it possible to examine combined bending-tension with the aid of (3.13), (3.14).

The resulting expressions (3.13) and (3.14) make it possible to study three-dimensional edge crack development and to make estimates of the fracture loads. By analogy with the preceding discussion, the initial development of the crack will generally be stable, which makes analysis of the limit states difficult. We make a simplifying assumption: the crack remains elliptical in the process of its stable development. This makes it possible to use (3.13) and (3.14) in the analysis of subcritical crack growth. According to (3.13) and (3.14), two cases are possible.

a) The initial crack has dimensions such that the coefficient of θ^2 is positive. Then the crack begins to develop along the edge adjacent to the surface of the strip and the crack depth b does not change (b/a decreases). Stable crack growth will continue until the stress intensity equalizes along the entire crack contour. It is obvious that the limiting state preceding transition into the dynamic regime is reached at the moment when the coefficient of θ^2 vanishes.

b) The initial crack has dimensions such that the coefficient of θ^2 is negative. Then the crack begins to develop depthward, while the surface length $2a$ of the crack does not change (b/a increases). Once again, as a result of this crack shape change there is a redistribution of the stress intensity along the crack contour leading to equalization. The limit state will be reached at the moment the coefficient of θ^2 vanishes, if we assume that the maximal value of b/h is less than 0.4. The latter case (b/h > 0.4) is close to the possibility of the existence of stable through cracks and is quite realistic for thinwall aircraft structures. However, it is not examined here.

We note that the fact of stable initial growth of part-though cracks has been known to the experimentalists for a comparatively long time. However, an adequately clear explanation has not been available (see, for example, [23]).

We shall present the final formulas obtained on the basis of (3.13) and (3.14), which define the critical crack dimensions and also the limit loads in the case of stable initial growth:

tension

$$\frac{b}{a} = \frac{2}{3} + \frac{b}{3h}, \quad \sigma_* = (0.68 \pm 0.04)\frac{K_{1*}}{\sqrt{b}} \quad \left(\frac{b}{h} \lessapprox 0.4\right) \tag{3.15}$$

bending

$$\frac{b}{a} = \frac{2}{3} - \frac{5b}{3h}, \quad M_* = 0.12\frac{K_{1*}h^2}{\sqrt{b}} f \quad \left(\frac{b}{h} \lessapprox 0.4\right) \tag{3.16}$$

$$f = \begin{cases} 1 \pm 0.1 & \text{for } 0 < b/h < 0.1 \\ 1.1 - 1.2b/h \pm 0.1 & \text{for } 0.1 < b/h < 0.4 \end{cases}$$

tension-bending

$$\frac{b}{a} = \frac{2}{3} + \frac{b(N-5)}{3h(N+1)}, \qquad N = \frac{0.17\sigma h^2}{M(1-0.5b/h)} \approx 1.1\frac{\sigma}{\sigma_m} \tag{3.17}$$

If $\sigma = (3 - 10)\,\sigma_m$ will be $b/a \approx 2/3$ and

$$K_{1*} \geqslant 1.4\sigma\sqrt{b}\left(1 + 0.5\frac{b}{h}\right) + 8.3\frac{M\sqrt{b}}{h^2}g$$

$$g = \begin{cases} 1 - b/h \pm 0.05 & \text{for } 0 < b/h < 0.2 \\ 0.8 \pm 0.05 & \text{for } 0.2 < b/h < 0.4 \end{cases}$$

Here σ_m is the maximal stress from the bending moment (in the extreme fiber); the equality sign in (3.15) corresponds to the fracturing load combination. The functions f and g approximate the more complex expressions obtained from (3.13) and (3.14). In the unstable crack growth case (3.13), (3.14) must be used from the very beginning.

Formulas (3.13)-(3.17) have acceptable accuracy for engineering calculations and encompass practically all possible cases of brittle fracture of thinwall vessels under pressure; substitution into these formulas of the values of σ_z in place of σ and M_x in place of M in accordance with (3.2) makes it possible to determine the external load parameters at the moment of fracture as a function of shell geometry, residual stresses, and the brittle fracture parameters (dimensions of initial defect and K_{1*}). If the working loads are specified, (3.15)-(3.17) serve as criteria for tolerable defect dimensions, magnitude of K_{1*} of the metal, shell geometry, and residual stresses.

Plasticity Correction. The exact calculation, which accounts for plastic effects and is based on examination of the hyperfine structure (see Section 1), involves tedious computational work and has not yet been carried out. Therefore, it is advisable in the initial stage to use the empirical plasticity corrections suggested by Irwin [23]. In application to the present problem, the correction amounts to increasing the dimension b by the magnitude Δb

$$\Delta b = \frac{p}{2\pi}\left(\frac{K_{1*}}{\sigma_{0.2}}\right)^2 \tag{3.18}$$

where p is a number selected to provide agreement with experiment (recommended values are: $p=1$ for through cracks in thin plates, $p=1/3$ for plane strain [1]).

It is assumed [1] that the correction shifts the limit of acceptability of linear fracture mechanics up to an average stress in the net cross section on the order of $\sigma_{0.2}$.

4. RELIABILITY OF STRUCTURE WITH CRACK

Comparative evaluation of structural operational reliability becomes of paramount interest for the high-strength materials with increasing danger of sudden brittle fracture. Two approaches are possible.

a) If the dimension b of the most hazardous initial crack-like defect can be detected by nondestructive testing with 100% probability, then the number χ serves as the reliability estimate [6] (for the same limit loads):

$$\chi = \frac{K_{1*}^{2}}{\sigma_{0.2}^{2} b} \qquad (b < 0.4h) \tag{4.1}$$

The larger the number χ, the more ductile is the fracture; the smaller the number χ, the closer the fracture is to the ideal brittle type.

b) Assume the quantities K_{1*}, $\sigma_{0.2}$, and in particular b, be known only with some probability. Then, for a valid selection of the safety margin we must first of all specify the structure operational confidence coefficient (say, 90, 95, or 99% – depending on the function performed by the component); then (3.13)-(3.18) are used to calculate the safety margin required to ensure the desired probability. Then comparison of the two structures (with the same safety margin and limit load) is made by comparing the distribution functions of the number χ.

In practice it may be more convenient to use the design safety margin and (3.13)-(3.18) to find the critical defect dimensions, and then on the basis of the available fracture statistics, or from analysis of the metallurgical and manufacturing processes during which defects are formed, evaluate the probability of supercritical cracks in the structure.

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